

Exercise 2-57

Given: A CO₂ tank connected to ambient air through a 5 cm I.D., 1 m long tube.

Required: (i) Daily rate of CO₂ lost by diffusion for T = 300 K, P = 1 atm.

(ii) Pressure differential to give an equal bulk flow of pure CO₂.

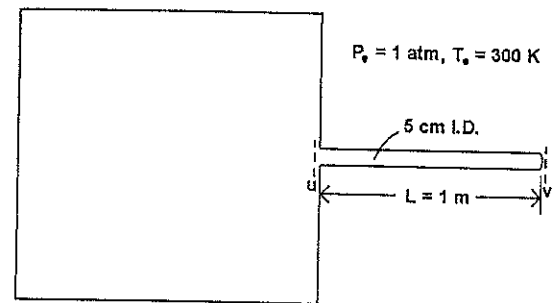
(iii) Mass average velocity and pressure gradient for part (i).

- Assumptions:
1. Steady, one-dimensional diffusion.
 2. Fixed CO₂ concentrations at each end of the tube.
 3. Ambient air contains 0.03% CO₂ by volume.

(i) For steady diffusion, assume Eq. (1.21b) applies

$$\dot{M}_1 = \frac{cD_{12}A}{L}(x_{1,u} - x_{1,v})$$

$$c = \frac{P}{RT} = \frac{1.0133 \times 10^5}{(8314)(300)} = 0.04063 \text{ kmol/m}^3$$



$D_{12} = 0.157 \times 10^{-4} \text{ m}^2/\text{s}$ from Table A.17a.

$A = (\pi/4)D^2 = (\pi/4)(0.05)^2 = 1.963 \times 10^{-3} \text{ m}^2$.

$\dot{M}_1 = (0.04063)(0.157 \times 10^{-4})(1.963 \times 10^{-3})(1 - 0.0003)/(1) = 1.252 \times 10^{-9} \text{ kmol/s}$

$\dot{m}_1 = (1.252 \times 10^{-9})(3600)(24)(44)(10^3) = 4.8 \text{ g/day}$

(ii) From Eq. (5.18) for fully developed laminar flow in a tube,

$$\frac{\Delta P}{L} = - \frac{8\mu u_b}{R^2}$$

At 300 K, 1 atm, Table A.7 gives $\rho = 1.788 \text{ kg/m}^3$, $\mu = 15.2 \times 10^{-6} \text{ kg/m s}$ for CO₂.

$u_b = \dot{m}_1/\rho A = (1.252 \times 10^{-9})(44)/(1.788)(1.963 \times 10^{-3}) = 1.570 \times 10^{-5} \text{ m/s}$

$\Delta P = -(8)(15.2 \times 10^{-6})(1.57 \times 10^{-5})(1)/(0.025)^2 = -3.05 \times 10^{-6} \text{ Pa } (\sim 10^{-11} \text{ atm})$

(iii) In (i) we assumed *equimolar counterdiffusion*: for each kmol of CO₂ diffusing out the tank, 1 kmol of air diffuses into the tank (since their concentration gradients are equal and opposite). The mole average velocity v^* is zero, and $N_1 = J_1 = -J_2 = -N_2$. The mass average velocity is not zero;

2-57 (continued)

$$\rho u_b A = \dot{M}_1 M_1 + \dot{M}_2 M_2 = \dot{M}_1 (M_1 - M_2)$$

$$u_b = (1.252 \times 10^{-9})(44 - 29) / (1.788)(1.963 \times 10^{-3}) = 5.35 \times 10^{-6} \text{ m/s}$$

at the tank end of the tube where $\rho = \rho_{\text{CO}_2}$. The corresponding pressure gradient required to balance viscous drag on the tube wall is

$$\frac{dP}{dx} = - \frac{8\mu u_b}{R^2} = - \frac{(8)(15.2 \times 10^{-6})(5.35 \times 10^{-6})}{(0.025)^2} = -1.04 \times 10^{-6} \text{ Pa/m}$$

Both the mass average velocity and the pressure gradient vary along the tube due to variations in ρ and μ of the mixture.

Comments: 1. The diffusion flow rate is very small.

2. In usual situations, bulk flows due to ambient pressure and temperature changes will be far more significant than diffusion flows.

3. The solution presented in part (i) is adequate for estimating approximate diffusion rates and pressure gradients. The conclusion in Comment No. 2 above is the significant result. However, the solution ignores the phenomenon of diffusion creep (or "slip"). When there is a concentration gradient along a wall, the no-slip velocity boundary condition for viscous flow is invalid: the flow "creeps" along the wall. An analogous phenomenon is the thermal creep due to a temperature gradient along a wall, discovered by Maxwell in 1878. Correct solutions to problems involving diffusion in capillary tubes and porous plugs are given in the following paper: Mills, A.F. "Diffusion Creep" *Int. Journal Heat Mass Transfer* 50: 5087-5098 (2007). Pedagogical aspects of this topic can be found in the paper "The Role of Diffusion Creep in Elementary Mass Transfer Analysis" which is posted on the faculty web page of A.F. Mills at UCLA-MAE.

Exercise 2-59

Given: A helium tank connected to ambient air through a 5 cm diameter, 1 cm thick perforated plate, with 50 μm diameter holes and an open area ratio of 0.7.

- Required:*
- Daily rate of He loss by diffusion for $T = 300\text{ K}$, $P = 1\text{ atm}$.
 - Pressure differential to give an equal bulk flow of pure helium.
 - Mass average velocity and pressure gradient for part (i).

- Assumptions:*
- Steady state.
 - Fixed He concentrations on each side of the disk.
 - One-dimensional diffusion and convection.

(i) For a time-wise steady state and no chemical reactions, the one-dimensional form of Eq. (2-16) is

$$\frac{d}{dz}(cv^*) = 0$$

Integrating

$$cv^* = N = \text{constant}$$

We will *assume* equimolar counterdiffusion. Then $N_1 = J_1^* = -J_2^* = -N_2$. (See Comment 2). From Eq. (2.15)

$$\frac{d}{dz}(J_1^*) = 0; \quad J_1^* = \text{constant} = -cD_{12} \frac{dx_1}{dz}$$

Integrating with $\dot{M}_1 = J_1^* A$ gives Eq. (1.21b),

$$\dot{M}_1 = \frac{cD_{12}A}{L}(x_{1,u} - x_{1,v}) \quad (1)$$

$$c = P/\mathcal{R}T = (1.0133 \times 10^5)/(8314)(300) = 0.04063 \text{ kmol/m}^3$$

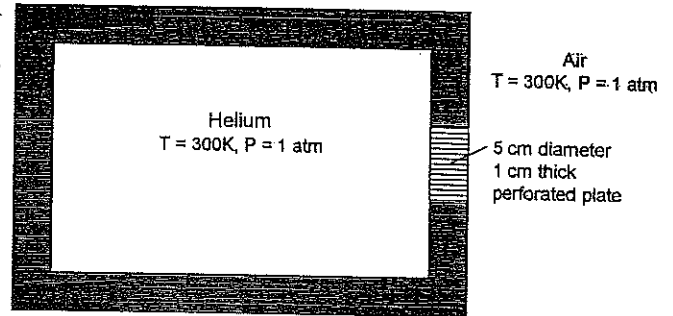
$$D_{12} = 0.713 \times 10^{-4} \text{ m}^2/\text{s}, \text{ from Table A.17a}$$

$$A = \epsilon_v A_{ff} = (0.7)(\pi/4)(0.05)^2 = 1.374 \times 10^{-3} \text{ m}^2$$

$$\dot{M}_1 = (0.04063)(0.713 \times 10^{-4})(1.374 \times 10^{-3})(1-0.000)/0.01 = 3.98 \times 10^{-7} \text{ kmol/s}$$

$$\dot{m}_1 = (3.98 \times 10^{-7})(4)(3600)(24)(10^3) = 137.6 \text{ g/day}$$

(ii) For the perforations, $L/D = (1 \times 10^{-2})/(50 \times 10^{-6}) = 200$. Assume fully developed laminar flow. From Eq. (5.18) of *Heat Transfer*,



2-59 (continued)

$$\frac{\Delta P}{L} = -\frac{8\mu u_b}{R^2} \quad (2)$$

At 300 K, 1 atm, Table A.7 gives $\rho = 0.1624 \text{ kg/m}^3$, $\mu = 20.1 \times 10^{-6} \text{ kg/m s}$ for helium. Thus,

$$u_b = \dot{m}_1 / \rho A = (3.98 \times 10^{-7})(4) / (0.1624)(1.374 \times 10^{-3}) = 7.13 \times 10^{-3} \text{ m/s}$$

$$\Delta P / L = -(8)(20.1 \times 10^{-6})(7.13 \times 10^{-3}) / (25 \times 10^{-6})^2 = -1.83 \text{ kPa/m}$$

$$\Delta P = -(1.83 \times 10^3)(0.01) = -18.3 \text{ Pa}$$

(iii) Although $v^* = 0$, the mass average $v = u_b$ is not-zero;

$$\rho u_b A = \dot{M}_1 M_1 + \dot{M}_2 M_2 = \dot{M}_1 (M_1 - M_2) \quad (3)$$

$$u_b = (3.98 \times 10^{-7})(4 - 29) / (0.1624)(1.374 \times 10^{-3}) = -4.46 \times 10^{-2} \text{ m/s}$$

at the tank end where $\rho = \rho_{\text{He}}$. At the other end, $\rho = \rho_{\text{air}} = 1.177 \text{ kg/m}^3$ and $u_b = -0.615 \times 10^{-2} \text{ m/s}$.

At the tank end of the disk the pressure gradient is obtained from Eq. (2) as

$$dP/dz = -8\mu u_b / R^2 = -(8)(20.1 \times 10^{-6})(-4.46 \times 10^{-2}) / (25 \times 10^{-6})^2 = 11.5 \text{ kPa/m}$$

and at the other end,

$$dP/dz = -(8)(18.43 \times 10^{-6})(-6.15 \times 10^{-3}) / (25 \times 10^{-6})^2 = 1.45 \text{ kPa/m}$$

- Comments:*
1. Check that a continuum model is adequate for part (i) by calculating the Knudsen diffusion coefficient, and comparing it to the ordinary diffusion coefficient.
 2. In usual situations, bulk flows due to ambient pressure and temperature changes will be far more significant than diffusion flows.
 3. The solution presented in part (i) is adequate for estimating approximate diffusion rates and pressure gradients. The conclusion in Comment No. 2 above is the significant result. However, the solution ignores the phenomenon of diffusion creep (or "slip"). When there is a concentration gradient along a wall, the no-slip velocity boundary condition for viscous flow is invalid: the flow "creeps" along the wall. An analogous phenomenon is the thermal creep due to a temperature gradient along a wall, discovered by Maxwell in 1878. Correct solutions to problems involving diffusion in capillary tubes and porous plugs are given in the following paper: Mills, A.F. "Diffusion Creep" *Int. Journal Heat Mass Transfer* 50: 5087-5098 (2007). Pedagogical aspects of this topic can be found in the paper "The Role of Diffusion Creep in Elementary Mass Transfer Analysis" which is posted on the faculty web page of A.F. Mills at UCLA-MAE.

Exercise 2-61

Given: A hydrogen tank connected to ambient air through a 5 cm diameter, 1 cm thick porous plug.

- Required: (i) Daily rate of H_2 loss by diffusion for $T = 300\text{ K}$, $P = 1\text{ atm}$.
 (ii) Pressure differential to give an equal bulk flow of hydrogen.
 (iii) Mass average velocity and pressure gradient through plug for part (i), and pressure differential.

- Assumptions: 1. Steady state.
 2. Fixed H_2 concentrations on each side of the plug.
 3. One-dimensional diffusion and convection.

(i) For a steady state and no chemical reactions, the one-dimensional form of Eq. (2-16) is

$$\frac{d}{dz}(cv^*) = 0$$

Integrating $cv^* = N = \text{constant}$

We will assume equimolar counterdiffusion. Then $N_1 = J_1^* = -J_2^* = -N_2$. (See Comment

2). From Eq. (2.15)

$$\frac{d}{dz}(J_1^*) = 0; J_1^* = \text{constant} = -cD_{12,\text{eff}}(dx_1/dz)$$

where $D_{12,\text{eff}} = (\epsilon/\tau)D_{12}$ following our analysis of porous catalysts in Section 1.6. Integrating

$\dot{M}_1 = J_1^* A_{\text{fr}}$ gives

$$\dot{M}_1 = \frac{cD_{12,\text{eff}}A_{\text{fr}}}{L}(x_{1,u} - x_{1,v}) \quad (1)$$

$$c = P/RT = (1.013 \times 10^5)/(8314)(300) = 0.0406 \text{ kmol/m}^3$$

$$D_{12} = 0.777 \times 10^{-4} \text{ m}^2/\text{s} \text{ (from Table A.17a)}$$

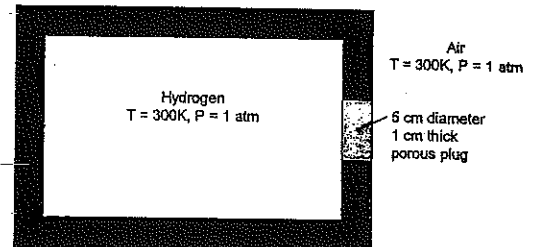
$$D_{12,\text{eff}} = (\epsilon/\tau)D_{12} = (0.8/1.8)(0.777 \times 10^{-4}) = 3.45 \times 10^{-5} \text{ m}^2/\text{s}$$

$$A_{\text{fr}} = (\pi/4)D^2 = (\pi/4)(0.05)^2 = 1.963 \times 10^{-3} \text{ m}^2$$

$$\dot{M}_1 = (0.0406)(3.45 \times 10^{-5})(1.963 \times 10^{-3})(1-0)/0.01 = 2.75 \times 10^{-7} \text{ kmol/s}$$

$$\dot{m}_1 = (2.75 \times 10^{-7})(2)(3600)(24)(10^3) = 47.5 \text{ g/day}$$

(ii) From Eq. (7.85) of *Heat Transfer*, Darcy's law for flow through a porous medium is



2-61 (continued)

$$\Delta P = \frac{\mu L \dot{m}}{\kappa \rho A_{fr}} \quad (2)$$

From Table A.7, $\mu = 8.93 \times 10^{-6}$ kg/s, $\rho = 0.0812$ kg/m³ for H₂ at 1 atm, 300 K. The Darcy permeability of the plug is 3.0×10^{-10} m² (given); thus

$$\Delta P = (8.93 \times 10^{-6})(0.01)(2.75 \times 10^{-7})(2)/(3 \times 10^{-10})(0.0812)(1.963 \times 10^{-3}) = 1.03 \text{ Pa}$$

(iii) Although $v^* = 0$, the mass average velocity is not zero;

$$\rho u_b A_{fr} = \dot{M}_1 M_1 + \dot{M}_2 M_2 = \dot{M}_1 (M_1 - M_2) \quad (3)$$

where u_b is a superficial velocity (based on plug frontal area). At the tank end, $\rho = \rho_{\text{He}}$ and

$$u_b = (2.75 \times 10^{-7})(2.29)/(1.963 \times 10^{-3})(0.0812) = -4.66 \times 10^{-2} \text{ m/s}$$

At the other end $\rho = \rho_{\text{air}} = 1.177$ kg/m³ and $u_b = -3.21 \times 10^{-3}$ m/s.

From Eq. (2), the pressure gradient at the tank end is

$$\frac{dP}{dz} = \left(\frac{\dot{m}}{\rho A_{fr}} \right) \frac{\mu_{\text{He}}}{\kappa} = \frac{u_b \mu_{\text{He}}}{\kappa} = \frac{(4.66 \times 10^{-2})(8.93 \times 10^{-6})}{3 \times 10^{-10}} = 1.39 \text{ kPa/m}$$

and at the other end where $\mu = \mu_{\text{air}} = 18.43 \times 10^{-6}$,

$$\frac{dP}{dz} = \frac{(3.21 \times 10^{-3})(18.43 \times 10^{-6})}{3 \times 10^{-10}} = 0.197 \text{ kPa/m}$$

Comments: 1. The diffusion flow rate is very small.

2. In usual situations, bulk flows due to ambient pressure and temperature changes will be far more significant than diffusion flows.

3. The solution presented in part (i) is adequate for estimating approximate diffusion rates and pressure gradients. The conclusion in Comment No. 2 above is the significant result. However, the solution ignores the phenomenon of diffusion creep (or "slip"). When there is a concentration gradient along a wall, the no-slip velocity boundary condition for viscous flow is invalid: the flow "creeps" along the wall. An analogous phenomenon is the thermal creep due to a temperature gradient along a wall, discovered by Maxwell in 1878. Correct solutions to problems involving diffusion in capillary tubes and porous plugs are given in the following paper: Mills, A.F. "Diffusion Creep" *Int. Journal Heat Mass Transfer* 50: 5087-5098 (2007). Pedagogical aspects of this topic can be found in the paper "The Role of Diffusion Creep in Elementary Mass Transfer Analysis" which is posted on the faculty web page of A.F. Mills at UCLA-MAE.